Eccentricity Evolution of Resonant Migrating Planets

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Received ______________________; accepted ______________________

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ABSTRACT

We examine the eccentricity evolution of a system of two planets locked in a mean motion resonance, in which the outer planet loses energy and angular momentum. The sink of energy and angular momentum could be either a gas or planetesimal disk. We show that the eccentricity of both planetary bodies can grow to large values, particularly if the inner body does not directly exchange energy or angular momentum with the disk. We analytically calculate the eccentricity damping rate in the case of a single planet migrating through a planetesimal disk. We present the results of numerical integrations of two resonant planets showing rapid growth of eccentricity. We also present integrations in which a Jupiter-mass planet is forced to migrate inward through a system of $5 - 10$ roughly Earth mass planets. The migrating planet can eject or accrete the smaller bodies; roughly 5% of the mass (averaged over all the integrations) accretes onto the central star. The results are discussed in the context of the currently known extrasolar planetary systems.

Subject headings: planetary systems—stars:

1. INTRODUCTION

The sixty or so extrasolar planetary systems known to date have revealed three striking features (for an up to date list of systems and their properties see http://www/exoplanets.org/ or http://www.obspm.fr/encycl/encycl.html). First, the distribution of orbital semimajor axes of the planets range from ~ 3 AU down to an almost incredible 0.038 AU. Second, most of the objects have high eccentricity by solar system standards, with a typical value being around $e = 0.4$, but ranging up to 0.927.

Third, the parent stars are highly metal rich, and appear to have accreted iron rich material after having reached the main sequence (Gonzalez et al. 2001; Santos et al. 2000; Laughlin 2000 ; Murray et al. 2001).

The simplest interpretation of the small orbits is that Jupiter-mass planets experience large scale migrations in some cases, but not in others; Jupiter falls into the latter class. There are currently two viable explanations for the migration, tidal interactions between the planet and the gas disk out of which it formed (Goldreich & Tremaine 1980; Lin et al. 1996), and gravitational interactions between the planet and a massive (1-5 Jupiter mass) planetesimal disk (Murray et al 1998).

The most straightforward interpretation of the high eccentricities, that they result from collisions or near collisions of two or more Jupiter-mass planets, is appealing, but require that most systems are dynamically unstable, in addition to undergoing migration. Furthermore, a recent exhaustive study of the problem indicates that the number of systems with low eccentricities is smaller than would be produced by collisions and scattering (Ford et al. 2001).

In this paper we investigate another possible mechanism for producing large eccentricities; resonant migration. We suppose that a Jupiter-mass planet is forced to migrate inward, either by tidal torques or by ejection of planetesimals, and that a second (possibly much less massive object) is in a mean motion resonance with the first. We further assume that the migration process does not significantly damp the eccentricity of the inner body. This could occur in migration in a gas disk if the inner disk manages to drain onto the central star while leaving behind the planets and a substantial outer gas disk. It would almost inevitably occur in migration through a massive planetesimal disk, since the planetesimals are likely to accrete into terrestrial mass or larger bodies; we show below by direct numerical integrations that these $1 - 50 M_\oplus$ bodies will be trapped into

mean motion resonances.

We show that the inward migration of two planets trapped in a mean motion resonance can produce eccentricities as high as 0.7. We also show that in the case of migration by planetesimal ejection, that the final state may or may not have two resonant planets. Whether the distribution of eccentricity with planetary mass and semimajor axis produced by such resonant migrations is consistent with the observed distribution is a question left for later work.

As a byproduct of our numerical simulations, we find that the fraction of planetesimal disk mass that accretes onto the star is likely to be much smaller than found in the work of (Quillen & Holman 2000); that work studied the accretion of massless test particles subject to gravitational perturbations from a migrating Jovian-mass planet. The authors found that of order half the mass in the disk would accrete. Using our more realistic, but less extensive integrations of massive planetesimals, we find a much smaller fraction ($\sim 5\%$) of the disk mass accreting onto the star in the early stages of the migration. (Another $\sim 5 - 10\%$ of the disk mass will fall on the star if the planet approaches within ~ 0.1 AU)(Hansen et al. 2001).

The remainder of the paper is organized as follows. In contrast to tidal torque migration the eccentricity evolution of a Jupiter-mass object migrating through a massive planetesimal disk has not been extensively studied. Section 2 gives a short derivation in the case that the migrating planet does not accrete a substantial fraction of the planetesimals. This is appropriate when the escape velocity from the surface of the planet is larger than the escape velocity at the orbital distance of the planet from the star. Section 2 describes the process of capture into resonance, and the evolution of the eccentricities of both resonant bodies as the migration proceeds. Section 4 presents the results of numerical integrations of two resonant bodies, with parameters appropriate for planetesimal migrations, as well

as integrations involving up to 11 planets. Section 5 gives a discussion of our results, and contrasts the two types of migration, while section 6 presents our conclusion.

2. MIGRATION AND ECCENTRICITY EVOLUTION

We examine the eccentricity evolution of a Jupiter-mass body migrating inwards due to the extraction of energy and angular momentum. The energy E_p and angular momentum L_p of the planet are given by

$$L_p = m_p \sqrt{G M_* a_p (1 - {e_p}^2)} \tag{1}$$

$$E_p = -\tfrac{G M_* m_p}{2 a_p} \tag{2}$$

Taking the time derivative of equation (1), we find the time variation of e_p in terms of the time variation of the planetary energy, assuming the planetary mass is fixed:

$$\frac{e_p}{1 - {e_p}^2} \frac{de_p}{dt} = -\frac{1}{2} \left(\frac{1}{E_p} \frac{dE_p}{dt} \right) \left[1 + 2 \left(\frac{E_p}{L_p} \frac{dL_p}{dE_p} \right) \right]. \tag{3}$$

The quantity

$$\beta \equiv \left[1 + 2 \left(\frac{E_p}{L_p} \frac{dL_p}{dE_p} \right) \right] \tag{4}$$

is a convenient measure of the rate at which the planetary eccentricity changes. Both E_p and dE_p/dt are negative for an inward migration, so e_p decreases if $\beta > 0$. Conservation of energy and angular momentum implies that $dL_p/dE_p = (dL/dE)_T$, where the latter quantity is the ratio of the rates at which angular momentum and energy are removed from the system by whatever process is driving the migration. The planetary eccentricity decreases as long as

$$\left(\frac{dL}{dE} \right)_T < -\frac{L_p}{2E_p} = \sqrt{1 - {e_p}^2} \Big/ n_p. \tag{5}$$

We now specialize to the case of planetesimal migration. To find dL_p/dE_p, we calculate the total change in E and L for a planetesimal of mass m (where $m << m_p$) from its initial

orbit, with semimajor axis a and eccentricity e, to the point at which it is ejected, then use conservation of energy and angular momentum. We note that this is not adequate for cases where the planet eats the planetesimal, since some orbital energy will be lost in the form of radiation in that case.

Figure (1) illustrates the constraints on the evolution of a planetesimal in the $E-L$ plane, in the case $e_p = 0.1$. The solid curved line corresponds to $e = 0$; planetesimal orbits must lie to the left of this line. In analyzing motion involving close encounters with a planet, it is useful to introduce the Jacoby parameter, $J = E - n_p L$, where n_p is the mean motion of the planet. The diagonal solid line corresponds to $J/m = -3/2$ (in units where $GM_* = a_p = 1$). We note that in order to be ejected from $a < a_p$ the asteroid must reach $J/m \geq -3/2$, since it must pass through $a = a_p$ with $e \geq 0$ to be ejected.

We will assume that the planetesimal is ejected with zero total energy; if it is ejected with a larger energy, the damping rate will be smaller than the estimate we obtain below. The initial energy and momentum of the planetesimal are given by expressions analogous to equations (1) and (2). Rather than calculating the change in L directly, we calculate the change in the Jacoby parameter; we do so because J is constant (in a statistical sense) during the planet crossing phase of the asteroid's evolution (Öpik 1976). To lowest order in m/m_p we have

$$\left(\frac{dL}{dE}\right)_T = \left[1 - \frac{dJ}{dE}\right] \Big/ n_p. \qquad (6)$$

If the planetesimal disk is originally cold ($e << 1$) and $e_p << 1$, few planetesimals will cross the orbit of the planet. However, planetesimals trapped in resonance with the planet will suffer chaotic perturbations which on average transfer angular momentum, but not energy, from the asteroid to the planet. This causes J to increase while leaving E fixed. Once enough angular momentum has been removed from the asteroid's orbit, the asteroid can suffer close encounters with the planet. We assume that the first close encounter removes

the asteroid from the resonance, while leaving J fixed. Subsequent encounters extract or supply energy and angular momentum to the asteroid in such a way as to leave J constant on average, as noted above. Eventually the planetesimal is ejected with $E \geq 0$; taking $E = 0$ we find

$$\frac{dJ}{dE} = 2\left(\frac{a}{a_p}\right)^{3/2}\left(\sqrt{1-e^2} - \sqrt{1-e_c^2}\right), \tag{7}$$

where $e_c \equiv a_p(1-e_p)/a - 1$ is the eccentricity at which the planetesimal just crosses the orbit of the planet. Note that $dJ/dE \geq 0$.

In arriving at equation (7) we have assumed that the final Jacoby parameter

$$J_f/m = J_c/m \equiv -\frac{1}{2a} - \sqrt{a(1-e_c^2)} > -3/2 \tag{8}$$

(we again use $GM_* = a_p = 1$). If the Jacoby parameter at planet crossing (J_c) is not larger than $-3m/2$, the planetesimal must diffuse to higher J in order to be ejected, since it has to get past the solid curve in Figure (1). Setting $J_f/m = -1.5$ we find

$$\frac{dJ}{dE} = 2\left(\frac{a}{a_p}\right)^{3/2}\left[\sqrt{1-e^2} + \frac{1}{2}\left(\frac{a_p}{a}\right)^{3/2} - \frac{3}{2}\left(\frac{a_p}{a}\right)^{1/2}\right]. \tag{9}$$

When $J_c/m > -3/2$ equation (7) should be used, while equation (9) is appropriate if $J_c/m < -3/2$.

Combining equations (3) and (6), the expression for the rate of change of the planet's eccentricity is

$$\frac{e_p}{1-{e_p}^2}\frac{de_p}{dt} = -\frac{1}{2}\left(\frac{1}{E_p}\frac{dE_p}{dt}\right)\left[1 - \frac{(1-dJ/dE)}{\sqrt{1-{e_p}^2}}\right] \tag{10}$$

It can be shown, using equations (7) and (9), that

$$\frac{dJ}{dE} \geq 1 - \sqrt{1-{e_p}^2}. \tag{11}$$

In other words, planetesimal migration, as described here, always damps the eccentricity of a single planet. We can estimate the value of dJ/dE when e and e_p are small; for example

for $J_c < -3/2$ (which requires $e \lesssim (3 + 2\sqrt{3})e_p$) equation (9) evaluated at the maximum $a/a_p = (1 - e_p)/(1 + e)$ gives

$$\frac{dJ}{dE} \approx \frac{3}{4}{e_p}^2 + \frac{3}{2}e_p e - \frac{1}{4}e^2. \tag{12}$$

For $e \approx e_p = 0.05$, $dJ/dE \approx 2{e_p}^2 \approx 0.005$. For smaller values of a this increases, as shown in Figure 2. Panel (a) in the Figure shows β and dJ/dE for a planet with $e_p = 0.05$ ejecting a planetesimal with initial $e = 0.05$, as a function of the initial a of the planetesimal. The relevant value of dJ/dE depends on the average initial a of the planetesimals that are ejected, i.e., it depends on which resonance is most actively ejecting objects. Early on in the evolution of the system we expect the planetesimal disk to be truncated inside the chaotic zone produced by the overlap of first order mean motion resonances (the "$\mu^{2/7}$ chaotic region"(Wisdom 1980)). Under those circumstances the relevant resonance is the 5/3, at $a/a_p \approx 0.71$; $dJ/dE \approx 0.066$ for that case. However, as the migration proceeds, material is supplied to the $\mu^{2/7}$ zone, and dJ/dE will be on average smaller, at least while $e_p << 1$. As e_p increases, equations (7) and (9) show that dJ/dE no longer increases as rapidly as e_p^2; it effectively saturates near $dJ/dE \approx 0.3 - 0.4$. Panel (b) shows β and dJ/dE in the case $e_p = e = 0.5$. Planetesimals never get the chance to reach the 5/3 resonance, since they become planet crossing at much smaller semimajor axis. The damping rate is of order 0.35.

We can calculate the circularization time for planetesimal migration, in terms of the migration time. Following the notation employed in satellite studies (Lissauer et al. 1984),

$$\frac{dE_p}{dt} = -(n_p T + H) \tag{13}$$

$$\frac{dL_p}{dt} = -T, \tag{14}$$

where T is the (average) torque exerted on the planet by the ejection of planetesimals, and H is responsible for removing energy from the radial motion of the planet, i.e., it damps the eccentricity. (Actually it would be better to use $dE_p/dt = -(n_p T/\sqrt{1 - {e_p}^2} + H)$, as will

become apparent). It follows from these equations that

$$H = -\beta \frac{dE_p}{dt}, \tag{15}$$

where we have replaced the missing factor of $\sqrt{1 - e_p{}^2}$. Using this in equation (13) we define the migration time

$$\tau_m \equiv \frac{GM_* m_p}{2a_p n_p T}(1 - \beta) \tag{16}$$

The circularization time τ_c is given by equation (3);

$$\tau_c \equiv \tau_m \frac{e_p{}^2}{\beta(1 - e_p{}^2)}. \tag{17}$$

For small e_p this reduces to $\tau_c \approx (2/3)\tau_m$; the circularization time is comparable to the migration time.

Note that in deriving equations (7) and (9) we have ignored the finite extent $D \equiv (m_p/3M_*)^{1/3} a_p$ of the planet's Hill sphere, the region over which the planet's gravity exceeds the tidal acceleration from the central star. For a Jupiter mass planet $D \approx 0.07 a_p$. Including the effect of the Hill sphere in our analysis effectively increase e_p to $e_p{}' = e_p + (m_p/3M_*)^{1/3}$; this is why we use $e_p = 0.1$ in Figure 1.

3. RESONANCE CAPTURE AND ECCENTRICITY EVOLUTION

We have seen that a single planet embedded in a planetesimal disk suffers eccentricity damping (it appears that a similar statement applies to a single planet in a gas disk (Papaloizou et al. 2001)). However, a Jupiter mass planet migrating through a disk of planetesimals will capture bodies into resonance; in the early stages this is how the migration proceeds. We show in this section that these resonant bodies tend to increase the eccentricity of the Jupiter mass planet; if $10 - 20$ Earth masses (denoted $M_\oplus$) are trapped into a resonance, then this resonant eccentricity driving exceeds the eccentricity

damping described in the previous section, and the eccentricity of the planet will increase as it migrates inward.

We begin by describing capture into resonance. We consider the gravitational interaction of two planets in orbit around a much more massive central body. For simplicity we consider only the planar problem. In the absence of dissipative effects the Hamiltonian describing the motion is

$$H = -\frac{\mu_1^2 m_1}{2L_1^2} - \frac{\mu_p^2 m_p}{2{L_p}^2} - \frac{G m_1 m_p}{a_1} \sum_{\mathbf{j}} \Phi_{\mathbf{j}}(a_1, a_p) e_1^{|j_3|} {e_p}^{|j_4|} \cos\left[j_1\lambda_1 - j_p\lambda_p + j_3\varpi_1 + j_4\varpi_p\right]. \quad (18)$$

Here $\mu_1 \equiv \mathcal{G}(M_* + m_1)$, where $\mathcal{G}$ is Newton's gravitational constant, and $L_1 = \sqrt{\mu_1 a_1}$, with similar definitions for the outer planet (labeled with a subscript p). The third term in equation (18) represents the mutual perturbations of the two planets. It produces variations in the orbital elements (a, e, and so forth) of order the planetary mass m_i.

The coefficient $\Phi \sim [a_1/(a_p - a_1)]^{|j_1 - j_p|}$ (Holman & Murray 1996). The integers j_i satisfy the relation $j_1 - j_p + j_3 + j_4 = 0$. Each cosine term in the sum is referred to as a resonant term or simply as a resonance. Resonances with $|j_1 - j_p| = q$ are proportional to q powers of eccentricity, and are said to be qth order mean motion resonances. The planets are said to be in resonance if one or more of the arguments of the cosines are bounded. Since $n_1 \equiv \left\langle \dot{\lambda}_1 \right\rangle$ (where the angle brackets refer to an average over a single orbit) and n_p are much larger than $\dot{\varpi}_1$ and $\dot{\varpi}_p$, the condition for resonance is roughly equivalent to

$$j_1 n_1 - j_p n_p = 0, \quad (19)$$

or $a_p/a_1 = (j_p/j_1)^{3/2}$. Throughout this section we ignore non-resonant terms of second order in the planetary masses.

Suppose that a_p/a_1 is initially larger than this resonant value, but that some dissipative process acts to reduce a_p while leaving a_1 unchanged. Then the torques represented by the resonant cosine term will increase, since $\Delta a \equiv a_p - a_1$ is decreasing and the torques are

proportional to $(a_1/\Delta a)^2$. Another way to say this is that the depth of the potential well represented by the resonant term is increasing, as is the width of the resonance. As a_p/a_1 passes through the resonant value $(j_1/j_p)^{2/3}$, the planets may be trapped into resonance. The torques represented by the resonant term in eqn. (18) will then transfer energy and angular momentum between the two planets in just such a way as to maintain the resonance while both bodies move toward the star (Goldreich 1965).

Capture is much less likely if the planets are moving away from each other, for example if a_1 is decreasing, or if both semimajor axes are decreasing but that of the inner planet decreases more rapidly; in that case the size of the resonance is decreasing, and the planets will usually pass through the resonance without being trapped.

Henceforth we will assume that the outer planet is moving toward the inner planet, resulting in capture into resonance.

Suppose for the moment that the inner planet has a sufficiently small mass that it cannot effectively scatter planetesimals, so that it does not lose energy or angular momentum directly to the planetesimal bath. (In the case of gas migration, we assume that the inner disk is non-existent). By virtue of its resonance interaction with the outer planet, it nevertheless does supply energy and angular momentum indirectly to material driving the migration. Another way to say this is that dE_p/dL_p no longer equals $(dE/dL)_T$, the quantity calculated for the case of planetesimal migration in the previous section. Here we calculate the relation between these two quantities that obtains when a second object of mass m_1, is in resonance with the Jupiter-mass object.

We relate dE_p/dL_p to $(dE/dL)_T$ using the conservation of energy and angular momentum. Conservation of energy gives

$$\frac{dE}{dE_p} = 1 + \frac{m_1}{m_p}\left(\frac{j_p}{j_1}\right)^{2/3}, \tag{20}$$

while conservation of angular momentum implies

$$\frac{dL_p}{dE_p} = \left(\frac{dL}{dE}\right)_T \left[1 + \frac{m_1}{m_p}\left(\frac{j_p}{j_1}\right)^{2/3}\right] - \frac{dL_1}{dE_p} \tag{21}$$

We also need

$$\frac{L_1}{L_p} = \frac{m_1}{m_p}\left(\frac{j_1}{j_p}\right)^{1/3}\sqrt{\frac{1-e_1^2}{1-{e_p}^2}}. \tag{22}$$

Using all these in (3) we find

$$\frac{e_p}{1-{e_p}^2}\frac{de_p}{dt} = -\frac{1}{2}\left(\frac{1}{E_p}\frac{dE_p}{dt}\right)\left[1 + 2\frac{E_p}{L_p}\left(\frac{dL}{dE}\right)_T\left(\frac{dE}{dE_p}\right)\right] + \frac{L_1}{L_p}\left(\frac{1}{L_1}\frac{dL_1}{dt}\right). \tag{23}$$

Note that the dE/dE_p factor multiplying $(dL/dE)_T$ is larger than one; it effectively increases $(dL/dE)_T$. From equation (5) we see that this will tend to increase e_p. The term proportional to dL_1/dt will tend to decrease e_p, but we shall see that its effect is smaller than that of the term involving dE/dE_p; this is the origin of the increase in eccentricity of two resonant bodies undergoing migration. From the expression for L_1, and using the resonance condition, we have

$$\left(\frac{1}{L_1}\frac{dL_1}{dt}\right) = -\frac{1}{2}\left(\frac{1}{E_p}\frac{dE_p}{dt}\right) - \frac{e_1}{1-e_1^2}\frac{de_1}{dt}. \tag{24}$$

As just noted the first term on the right will tend to damp the eccentricity of the outer planet; the second term on the right will also damp e_p as long as $de_1/dt > 0$. Combining the last two equations we find

$$\begin{aligned}\frac{e_p}{1-{e_p}^2}\frac{de_p}{dt} &= -\frac{1}{2}\left(\frac{1}{E_p}\frac{dE_p}{dt}\right)\Bigg\{\left[1 + 2\frac{E_p}{L_p}\left(\frac{dL}{dE}\right)_T\right]\left(\frac{dE}{dE_p}\right) \\ &\quad -\frac{m_1}{m_p}\left(\frac{j_1}{j_p}\right)^{1/3}\frac{1}{\sqrt{1-{e_p}^2}}\left[\left(\frac{j_p}{j_1}\right)\sqrt{1-{e_p}^2} - \sqrt{1-e_1^2}\right]\Bigg\} \\ &\quad -\frac{m_1}{m_p}\left(\frac{j_1}{j_p}\right)^{1/3}\sqrt{\frac{1-e_1^2}{1-{e_p}^2}}\frac{e_1}{1-e_1^2}\frac{de_1}{dt},\end{aligned} \tag{25}$$

where we have added and subtracted $(m_1/m_p)(j_p/j_1)^{2/3}$ inside the curly brackets.

We identify

$$\left(\frac{dE_p}{dt}\right)_T \equiv \left(\frac{dE_p}{dt}\right)\left(\frac{dE}{dE_p}\right) \tag{26}$$

as the rate at which the expulsion of planetesimals removes energy from the outer planet. With this identification, and recalling equation (3), it becomes clear that the first term in curly brackets in equation (25) is the time rate of change of e_p due to the expulsion of planetesimals. The final result is

$$\begin{aligned}\frac{e_p}{1-{e_p}^2}\frac{de_p}{dt} \quad = \quad & -\frac{1}{2}\left(\frac{1}{E_p}\frac{dE_p}{dt}\right)_T \left\{1 - \frac{(1-dJ/dE)}{\sqrt{1-{e_p}^2}}\right. \\ & -\frac{m_1}{m_p}\left(\frac{j_1}{j_p}\right)^{1/3}\frac{1}{\sqrt{1-{e_p}^2}}\left[\left(\frac{j_p}{j_1}\right)\sqrt{1-{e_p}^2} - \sqrt{1-e_1^2}\right] \Bigg/ \left.\left[1+\frac{m_1}{m_p}\left(\frac{j_p}{j_1}\right)^{2/3}\right]\right\} \\ & -\frac{m_1}{m_p}\left(\frac{j_1}{j_p}\right)^{1/3}\sqrt{\frac{1-e_1^2}{1-{e_p}^2}}\frac{e_1}{1-e_1^2}\frac{de_1}{dt}\end{aligned} \tag{27}$$

Note that this expression reduces to equation (10) when $m_1 \to 0$.

Equation (27) has two undetermined quantities (de_p/dt and de_1/dt). Bodies that are trapped in a mean motion resonance typically have their apsidal lines locked as well, so that $\dot{\varpi}_1 = \dot{\varpi}_2$. Using the equations of motion for ϖ_1 and ϖ_2, we can find a relation between e_1 and e_p that depends on the precession rates of the apsidal lines. The latter are determined both by the mutual perturbations of the two planets, and by the distribution of mass in the planetesimal (or gas) disk. Given the current state of both observations and theory, we feel that a detailed calculation is not justified.

In the appendix we present another derivation in which we allow for the possibility that the inner, less massive planet also loses energy and angular momentum to the sink of energy and angular momentum.

We proceed to examine some limiting cases. First, suppose that no tides act on the inner planet, and that the tides acting on the outer planet keep that planet's orbit circular.

Then

$$\frac{e_1}{\sqrt{1-e_1^2}}\frac{de_1}{dt} \approx -\left(\frac{d\ln a_p}{dt}\right)_T \left[\frac{j_p}{j_1} - \sqrt{1-e_1^2}\right] \Big/ 2\,(dE/dE_p)\,. \tag{28}$$

Since a_p is decreasing, the right hand side is positive, and e_1 will grow. Assuming $j_p/j_1 >> 1$, we find

$$e_1 \approx \sqrt{1-(1-\gamma_1 \ln a_{1,i}/a_1)}, \tag{29}$$

where $a_{1,i}$ is the semimajor axis of the inner body when it is captured into resonance, $\gamma_1 = j_p/2j_1$, and we have neglected the initial value of e_1.

Now suppose that no tides act on the inner body, and that the migration has proceeded far enough that e_1 has grown to the point that de_1/dt is small. Then we can find the equilibrium value for e_p by setting the terms in the curly brackets equal, assuming that $\beta = 2{e_p}^2/3$;

$$e_{p,max} \approx \sqrt{\frac{2m_1 a_p}{3(m_p a_1 + m_1 a_p)}}, \tag{30}$$

where we neglect $\sqrt{(1-e_1^2)/(1-{e_p}^2)}$ compared to j_p/j_1. For gas-disk migration, the ratio of τ_c/τ_m would enter in the expression for β. For two equal mass bodies in a 4/1 resonance this is about 0.7. Even for $m_1/m_p = 0.1$ (a $30M_\oplus$ inner planet) the equilibrium eccentricity is 0.41. However, note that this expression is actually an underestimate in the case of planetesimal migration, since β, which is a measure of the damping due to the migration process, does not scale as ${e_p}^2$ for e_p as large as 0.4; β actually grows less rapidly, meaning that the damping is not as efficient as equation (30) assumes.

We can relate the eccentricity to the distance migrated when $m_p >> m_1$. Define

$$\gamma \equiv \frac{m_1}{m_p}\left(\frac{j_p}{j_1}\right)^{2/3}. \tag{31}$$

The evolution of the eccentricity is then described by

$$\frac{1}{1-{e_p}^2}\frac{d{e_p}^2}{dt} \approx -\frac{1}{2}\left(\frac{1}{E_p}\frac{dE_p}{dt}\right)_T [\beta-\gamma] \tag{32}$$

We have assumed that $j_p/j_1 >> \sqrt{1-e_1^2}$, and neglected terms second order in m_1/m_p. The eccentricity of the outer planet will grow indefinitely providing $\gamma > \beta$.

Now suppose that β is independent of e_p; this is not true when e_p is small, but for small e_p γ can be much larger than β; for $e_p \approx 0.2$ or larger β is roughly constant. In that case we can integrate equation (32) to find the final eccentricity e_{2f} of the outer body,

$$e_{2f} \approx \sqrt{1 - \left(\frac{a_{2f}}{a_{2i}}\right)^{\gamma-\beta}}, \tag{33}$$

where a_{2i} is the semimajor axis and eccentricity of the outer planet when it enters the resonance, and a_{2f} is the semimajor axis when the planet either stops migrating or leaves the resonance. We have assumed that the initial eccentricity $e_p << 1$; if it is not, the final eccentricity will be larger.

The numerical work described below shows that a Jupiter-mass object can capture smaller bodies into resonances ranging from the 2/1 to the 4/1; we have even seen captures into the 11/2 resonance. In a scenario where the Jupiter-mass body migrates through a planetesimal disk having a comparable mass, we expect resonance capture of terrestrial bodies with masses ranging from 1 Earth mass ($M_\oplus$) up to $30M_\oplus$ or more; an extreme upper limit might be of order $50M_\oplus$, corresponding to about 10% of the disk mass. The plausible range for γ is then $5 \times 10^{-3} - 0.4$. For migration in a planetesimal disk we expect β to be in the range 0.01 (for $e_p << 1$) to 0.3 (for $e_p \gtrsim 0.5$). Taking 1 AU as a representative value for a_{2f} (although some extrasolar planets are in much smaller orbits), with $a_{1f} \sim 5 - 10$ AU, we find final eccentricities in the range $0.1 - 0.6$, with $e_p \approx 0.45$ being a typical value.

4. Numerical Results

We employ a Bulirsch-Stoer integrator with a variable step size. We require that at each time step the relative accuracy of the integration (as measured in phase space) be

10^{-12}. Typical orbital times are of order one to ten years, while the integrations can extend up to 10^8 yrs. In test runs where no energy is removed from the planet, the largest variation in total energy is typically less than a part in 10^9. In most of the runs reported on here we remove energy and angular momentum from the largest planet; the variations in energy and angular momentum from the expected amounts are similarly small.

In runs with multiple massive planets collisions often occur. We assume that the smaller planets are rocky bodies with bulk densities of 3 g/cm^3, that collision occur when planets are within two times the sum of the planetary radii (to allow for capture by tidal disruptions) and that the captures are completely inelastic with no loss of mass to small fragments. This is reasonable for collisions involving the more massive objects.

We assume that the most massive body ("Jupiter") migrates inward by ejecting numerous planetesimals from the system (Murray et al. 1998). We simulate this by extracting energy and angular momentum from the orbit of Jupiter. We implement this numerically as follows.

The energy and angular momentum of the planet are given by

$$E = \frac{1}{2} m_p (v_x^2 + v_y^2) - \frac{GM_* m_p}{r} \tag{34}$$

and

$$L = m_p (x v_y - y v_x) \tag{35}$$

where v_x and v_y are the velocity of the planet (we suppress the subscript p for ease of reading). Recall that we assume the planet and planetesimal are coplanar. Taking time derivatives, we invert to find

$$\begin{aligned} \frac{dv_x}{dt} &= \left(\frac{1}{E_p}\frac{dE_p}{dt}\right) \frac{E_p}{m_p \mathbf{r}\cdot\mathbf{v}} \left[x + \frac{v_y}{n_p}\sqrt{1-{e_p}^2}(\beta-1)\right] \\ \frac{dv_y}{dt} &= \left(\frac{1}{E_p}\frac{dE_p}{dt}\right) \frac{E_p}{m_p \mathbf{r}\cdot\mathbf{v}} \left[y - \frac{v_x}{n_p}\sqrt{1-{e_p}^2}(\beta-1)\right], \end{aligned} \tag{36}$$

where we have used equation (4). We assume that the close encounters that lead to changes in the planetesimal's orbital elements occur on times much shorter than the orbital period, so that we can take the position of the planet to be fixed.

Using these equations for numerical work is problematic, since the vector dot product in the denominator vanishes at peri- and apoapse. We regularize the equations by multiplying the right-hand side by $2\sin^2 f$, where f is the true anomaly.

The terrestrial mass objects in our simulations do not have high enough escape velocities to efficiently eject smaller bodies, so we do not force them to migrate.

We have integrated the equations of motion for the two body problem (the star and a massive planet) modified to account for the drag imposed by the ejection of planetesimals, as given in equation (36), regularized as noted above. The eccentricity e_p and semimajor axis a_p decay as expected. We discuss the behavior of systems with more than one massive planet in the following subsections.

4.1. TWO MASSIVE BODIES

To test the basic idea that migration of two resonant bodies will induce the growth of eccentricity, we have started two planets just outside resonance, and applied the "tides" described in the previous paragraphs. An example is shown in Figure (3). The inner body has a mass of $20M_\oplus$ and the outer body has a mass equal to that of Jupiter ($\approx 318M_\oplus$). The upper plot shows the semimajor axes of both bodies as a function of time, while the lower plot shows the eccentricities. Energy and angular momentum were removed only from the outer body. The resonance interaction forces the inner body to migrate inward as well, as can be seen in the figure. One can also see from the figure that the eccentricity of both bodies increased.

The prediction based on (33) for the eccentricity of the outer, more massive body is too large for a_p near the initial value, which is consistent with the fact that we neglected the rather rapid variation of e_1; as the migration proceeds the prediction becomes more accurate. By the end of the integration equation (33) is a good approximation.

4.2. MULTIPLE MASSIVE BODIES

The planetesimal disk we postulate is very massive, $1 - 3$ Jupiter masses. It is likely that multiply bodies with masses comparable to or larger than that of the Earth are likely to form in such a massive disk. As a first step toward a realistic simulation of a migration in such a disk, we have run a number of cases involving five to ten roughly Earth mass bodies placed on orbits with random semimajor axes and small eccentricities inside the orbit of a Jupiter-mass planet. We then force the Jupiter mass body to migrate inward toward the Earth mass planets.

Figure (4) shows the result of one such integration. We started five bodies with masses randomly distributed between 0.3 and $10M_\oplus$ with semimajor axes between 0.5 and 4 AU. As Jupiter migrated inward, three of the small planets merged to form a $6.3M_\oplus$ planet, one small planet crashed into Jupiter, and one small planet was ejected. Both the latter two events illustrate the migration mechanism we are postulating.

After the three small planets merged to form a $6.3M_\oplus$ body, the resulting planet was captured into the 3/1 mean motion resonance with Jupiter. Subsequently the eccentricities of both bodies increased (we employed a rather low value of $\beta/{e_p}^2$ for this run). Both planets migrated inward until the inner planet struck Jupiter, when Jupiter was at 0.12 AU, with an eccentricity of 0.4.

In this run most of the mass in the disk actually accreted onto the Jupiter mass planet.

Part of the reason for this is that the smaller planets were started at small radii, where the escape velocity from the system exceeded the escape velocity from the Jupiter mass planet; in that case we expect that most planetesimals will accrete onto Jupiter rather than be ejected from the system. Only one body was ejected. On the other hand, no bodies hit the star.

The latter result is representative of most of our runs; the fraction of mass accreting onto the star is small, $\sim 5\%$. The fraction ejected varies with the initial semimajor axis and the mass of the Jupiter mass body; both larger initial a_p and larger m_p produce a larger fraction of ejected bodies (relative to bodies accreted onto the massive planet). These rather low accretion fractions are in stark contrast to those found by (Quillen & Holman 2000). The difference appears to be that we employ massive planetesimals, while their simulations employed only test particles, which did not interact with each other. In our simulations only the second most massive body (the first being "Jupiter") remains for long in a resonance; this second most massive body lords it over his smaller brethren, kicking them out of nearby resonances they might like to occupy. This tends to prevent the smaller objects from reaching the extremely high eccentricities (> 0.9) needed to strike the star.

In other runs the final state includes two planets in a mean motion resonance. Since we start with such low planetesimal masses and numbers, the mass ratio was always large. However, we expect that if we allow larger terrestrial bodies to grow, that we may well find final states with mass ratios nearer to unity. Finally, we note that recent simulation of planetesimal migration show that two Jupiter-mass object placed in a planetesimal disk will on some occasions migrate toward each other (Hansen et al. 2001). This could lead to resonance capture followed by inward migration. Interactions between the massive bodies and the planetesimal disk would likely tend to damp the eccentricity of both bodies, but equation (A10) indicates that as long as the outer body lost energy at a higher rate, the

eccentricities of both bodies would grow.

5. DISCUSSION

The mechanism we have proposed for the growth of eccentricity with inward migration is essentially the same as that used to explain the non-zero eccentricities of the inner Jovian satellites. It is well understood and quite robust. It does not rely on the details of the migration mechanism; in the case of the Jovian satellites the migration is a result of the tidal bulge raised by Io on Jupiter; the bulge exerts a torque on Io which transfers energy from Jupiter's spin to the orbit of Io. Io in turn exerts, through a 2/1 mean motion resonance, a torque on Europa. In the satellite case the eccentricity damping, produce by tidal flexing in both satellites as they oscillate from peri- to apo-Jove, is very strong. This limits the eccentricity to a value which, while small, is sufficient to dissipate enough energy to power the volcanism on Io.

In previous sections we have examined resonant eccentricity growth in the context of planetesimal migration, but it can work in the context of migration due to tidal torques imposed by a gas disk as well. Suppose that two Jupiter-mass bodies embedded in a gas disk are locked in a mean motion resonance. Suppose that the gas between the planets is removed, as numerical integrations indicate (Bryden et al. 2000). The gas inside the orbit of the inner planet will accrete onto the star, possibly with some fraction being removed by a disk or stellar wind. The planets are likely to follow the inner disk inward; if they do not, the normal viscous spreading of the inner disk would move the outer edge of the disk outward, until it experiences tidal torques from the inner planet; this interaction would produce a back reaction which would tend to damp the eccentricity of that planet; large planetary eccentricities are unlikely to arise in that case.

However, it may be possible that the inner disk drains onto the star, leaving the planets behind. This could occur, for example, if the outer disk had a mass only slightly larger than that of the planets (Nelson et al. 2000). Then only the outer planet will experience significant tidal torques, since the first order resonances of the inner planet lie in the region between the planets that is depleted of gas. Both planets will then migrate inward, and eccentricity of the inner planet will grow, since it does not experience much eccentricity damping. This is exactly analogous to the planetesimal migration described above, and the expressions we have given will describe the growth and equilibrium values of the eccentricity once the appropriate eccentricity damping rate for the outer planet is introduced (see, e.g., Goldreich & Tremaine 1980).

The tidal torque scenario also requires that the outer planet have a mass sufficient to open a gap in the gas disk. If it does not, then both bodies will experience tidal torques, which tend to damp eccentricity rather strongly. An approximate criterion for gap formation is (Lin & Papaloizou 1986)

$$\frac{m}{M_*} \gtrsim 40\alpha(c_s/v_k)^2, \tag{37}$$

where α is the the (Shakura & Sunyaev 1973) viscosity parameter, c_s is the sound speed in the gas disk, and v_k is the Keplerian rotation velocity. If the disk is ionized, then the Balbus-Hawley instability (Balbus & Hawley 1991; Hawley & Balbus 1991) is likely to produce a rather larger (effective) α, of order 0.5. The planet must then have a mass of order 200 Jupiter masses (m_J) in order to open a gap. At small orbital radii $a \lesssim 0.1$ AU the disk will be ionized (Gammie 1996). This suggests that if the capture into resonance occurs at very small radii, or if the planets migrate to very small radii, the eccentricity of both bodies will be damped.

However, at larger radii protoplanetary disks are believed to be substantially neutral, so that they are not subject to the Balbus-Hawley instability (Gammie 1996). If so, they

will likely have a small effective viscosity, and equation (37) predicts that gap opening will occur for small (subJovian) mass planets. In terms of α currently favored values are in the range $\alpha \approx 10^{-4}$ to 10^{-2}. The latter value yields a mass for a gap clearing planet of about $2m_J$. Smaller values of α would yield smaller masses, but it is believed that in that case a second criterion is relevant, namely $m/M_* > 3(c_s/v_k)^2$ (Papaloizou & Lin 1984). At 5 AU this yields $m \gtrsim m_J$. Thus the eccentricity of the inner planet will only be excited if both objects are of roughly Jupiter mass, assuming the migration is driven by tidal torques in a gas disk.

Another constraint is that the migration torque not exceed the resonant torque. If it does, the resonance will be broken, and the eccentricity of the outer body will drop. The outer body may then migrate inward, perhaps to be caught into a stronger resonance. This constraint is likely to be important in the capture phase, particularly in a migration produced by tidal torques. In that situation, both planets are likely to have very small eccentricities. The tidal torque is given by (Ward 1997)

$$T_{disk} \approx \left(\frac{GM_* m_p}{2a}\right) \frac{m_p}{M_*} \frac{M_{disk}}{M_*} \left(\frac{v_k}{c_s}\right)^3, \tag{38}$$

assuming that the outer planet does not open a gap. For a disk of mass $M_{disk} = 10^{-2} M_*$ and a planet at 5 AU, this is about $50(GM_* m_p/2a)$. If the outer planet is massive enough to open a gap, the torque is set by the viscosity in the gas disk,

$$T_{gap} \approx \left(\frac{GM_* m_p}{2a}\right) \alpha \left(\frac{c_s}{v_k}\right)^2. \tag{39}$$

This is much smaller than the torque in the gapless case, $T_{gap} \approx 10^{-5}(\alpha/10^{-2})(GM_* m_p/2a)$. The resonant torque is

$$T_{res} \approx \left(\frac{GM_* m_p}{2a}\right) \frac{m_1}{m_p} \left(\frac{e a_1}{a_1 - a_p}\right)^q, \tag{40}$$

where e is the larger of the eccentricities of the two planets. This eccentricity is likely to be small; if we take $e \approx 0.01$ then the resonant torque is $T_{res} \approx 10^{-2}(m_1/m_p)(GM_* m_p/2a)$ for

the 2/1 first order resonance; near the inner edge of the gap (where $a/\Delta a \approx 10$) this will rise by about 10.

If both bodies are of roughly Jupiter mass, capture is possible into first or second order resonances, that is, resonances with $|j_1 - j_p| = q$, where $q = 1$ or 2, since T_{gap} is the appropriate torque to use. This case may arise in the scenario mentioned above, where gas caught between two giant planets can leak out over several hundred orbital periods. The tidal torques from the gas inside the inner planet will then tend to push it outward, while the gas outside the outer planet will tend to push it inward; capture into the 2/1 or possibly the 3/1 mean motion resonance could then occur.

If the outer body has a mass substantially smaller than m_J, it will not open a gap in the gas disk. If it has a mass comparable to or larger than $1M_\oplus$, the hydrodynamic drag it experiences will be much smaller than the tidal torques. It will undergo rapid (Type I in the notation of Ward) inward migration, easily passing through any mean motion resonances (see equation 38). According to Ward, the time for this inward migration will be less than 10^5 years. The inward migration will not halt until the outer body enters the gap produced by the inner, Jovian mass object. It will then experience a torque similar to that felt by the inner, Jupiter mass body, and both bodies will migrate inward without a substantial change in their eccentricity.

Since the initial inward migration is so rapid, it seems unlikely that an outer planet with initial $m_1 \lesssim 10M_\oplus$ will be able to accrete sufficient solid material to trigger the accretion of gas before it enters the gap produced by the inner planet. Once it enters the gap, the outer planet could grow by eating other, inward migrating bodies, a la the scenario proposed by Ward (1997) for explaining the very short period Jupiter mass objects. However, unlike Ward's case, the outer planet cannot emerge far enough from the inner edge of the outer disk that its 2/1 resonance leaves the disk, slowing the inward migration;

the inner planet is in the way. Given the mismatch between the tidal and resonant torques, it seems likely that the smaller planet will be subsumed by the more massive body.

There may be ways to distinguish migration by tidal torques and migration by ejection of planetesimals. Planetesimal migration is likely to produce dynamically isolated (although possibly not unaccompanied) Jupiter mass bodies in small, eccentric orbits; as we have seen, the small mass inner body responsible for driving the eccentricity up to large values is often ingested into the star or the Jupiter mass object.

In those cases where the inner body survives the migration process, it should be possible to detect it with high precision radial velocity observations. In some cases the inner body may have a large mass, since planetesimal migration involving more than one Jupiter mass object sometimes produces a convergence of the semimajor axes of two bodies (Hansen et al. 2001). This might produce systems like those recently discovered around GJ876 (Marcy et al. 2001). Alternately, such a system could be the outcome of the migration of two Jupiter-mass planets in a gas disk.

Resonant migration in a gas disk is less likely to produce a single body in a moderately eccentric orbit than is migration in a planetesimal disk; in the former case both bodies are likely to be deep in resonance, and hence protected from close encounters and the subsequent carnage. If they do suffer close encounters, merger rather than ejection or accretion onto the central star is the likely result. In a planetesimal migration, the low mass of the inner planet combined with the frequency of close encounters with numerous smaller bodies tends to keep the amplitude of libration large. We have seen several cases in our numerical integrations where the inner planet collides with the Jupiter mass body, or with the central star.

There are several systems which have low mass planets in highly eccentric orbits, including HD108147 ($M \sin i = 0.35 M_J$, $a = 0.098$ AU, $e = 0.56$, and $K = 37$m/s), HD83443

($M \sin i = 0.17 M_J$, $a = 0.174$ AU, $e = 0.42$, and $K = 14$m/s) (the system contains at least two planets, currently not dynamically linked), and HD16141 ($M \sin i = 0.22 M_J$, $a = 0.351$ AU, $e = 0.28$, and $K = 10.8$m/s). The first system is particularly interesting as a test of the type of migration involved, assuming the eccentricity is produce by resonant migration. The putative resonant planet must have a mass less than about 1/3 the mass of the detected planet, in order to escape detection (since the survey is clearly capable of finding planets with $K \approx 10$ m/s); this would give $M \sin i \approx 0.1 M_J$. For a typical inclination we would expect a mass of about of about $60 M_\oplus$. Whether such a low mass object could open a gap in a gas disk is an interesting question. We note that the planet in HD108147 is near the radius at which the Balbus-Hawley instability is believed to operate. In a gas disk, a planet in this region would be subject to rapid eccentricity damping. As radial velocity surveys improve below the 10m/s level, the discovery of even lower mass objects in eccentric orbits would indicate that some mechanism other than resonant migration in a gas disk was operating to produce the high eccentricities.

The fact that our simulations show accretion of planetesimals onto the star suggests another way to distinguish the two scenarios. The planetesimal migration is inevitably accompanied by the accretion of $\sim 5\%$ of the planetesimal disk mass onto the star; we see this even in simulations in which we halt the migration at large semimajor axis. Since the mass of the disk is of order $300 - 600 M_\oplus$, this amounts to $\sim 20 M_\oplus$ of rocky material accreted onto the star. This material will include $5 - 7 M_\oplus$ of iron, altering the apparent metallicity of the parent stars dramatically. Note that Jupiter contains only about $2 M_\oplus$ of iron.

Moderate period (longer than 40 day period) systems produced by gas migration are unlikely to pollute their parent stars so dramatically; there is no reason to expect that such systems have a few hundred Earth masses of rocky material lying around. The parent

stars could accrete metal rich Jupiter mass bodies after reaching the main sequence; but again there is no reason to expect that every moderate period system did so, when it is known that most stars that lack such companions did not (Murray et al. 2001). We say this because planets with 40 day or longer periods and moderate eccentricities (~ 0.5 or less) are dynamically uncoupled from very short period planets (four days or so), and so are unlikely to cause them to fall onto the star.

6. CONCLUSIONS

We have described how two resonant planets undergoing inward migration can reach eccentricities of order 0.7. The eccentricity growth is largest when the inner planet is not subject to eccentricity damping. Such a situation may arise either in planetesimal migration, or in migration driven by tidal torques in which the inner gas disk has been removed by accretion or mass loss in a wind. We have presented expressions for the equilibrium eccentricity, when it exists, and for the relation between e and the initial and final semimajor axis of the resonant planets.

We have presented numerical integrations showing that in some cases a planetesimal migration will produce a single Jupiter mass object with a large eccentricity; in other cases the Jupiter-mass object may be accompanied by a resonant object with a similar mass, or by a Neptune-mass companion. In the case of a Neptune-mass body the inner companion will have a very large eccentricity.

We have also described integrations involving 10 or more roughly Earth mass bodies, together with a Jupiter mass planet; the latter is forced to migrate inward, with the migration process tending to damp its eccentricity. It typically captures one or more of the less massive bodies into mean motion resonance. Usually only the second most

massive planet (which may not be the body that was originally second most massive behind "Jupiter") survives long in resonance. In fact, this second most massive body tends to accrete its smaller companions.

We have suggested two possible ways to tell the difference between systems produced by planetesimal or gas migration. First, if there exist a large number of systems having an eccentric planet with either no resonant companion, or with only very low mass (no-gap opening) companions, a finding requiring very high precision radial velocity measurements, it would strongly suggest that resonant migration by tidal torques was not responsible. Second, the planetesimal migration picture predicts that several Earth masses of iron will be accreted in the migration process, well after the central star has reached the main sequence. The tidal torque scenario is mute regarding this point; accretion of material after the gas disk vanishes, and closely correlated with the presence of Jupiter-mass planets, is not a natural feature.

A. Appendix

To derive the expression for the variation of the eccentricities of two planets caught in a mean motion resonance when one or both are subject to dissipative forces, we examine the equations describing the evolution of energy and angular momentum, subject to the constraint that the planets are locked in a resonance (Lissauer et al. 1984; Gomes 1998).

We start with the energy, which is given by

$$E = -\frac{GM_* m_1}{2a_1} - \frac{GM_* m_p}{2a_p}. \tag{A1}$$

We assume $a_1 < a_p$. It will prove useful to employ the variables $\mathcal{L}_i \equiv \sqrt{GM_* a_i}$; then the resonance condition (19) implies

$$\frac{d \ln \mathcal{L}_1}{dt} = \frac{d \ln \mathcal{L}_p}{dt}. \tag{A2}$$

This equation is accurate only in an average sense; on times shorter than the libration time of the resonance it is violated.

We assume that some dissipative force removes both energy and angular momentum from the orbits;

$$\frac{dE}{dt} = \left(\frac{dE_1}{dt}\right)_T + \left(\frac{dE_p}{dt}\right)_T, \tag{A3}$$

where the subscript T (for "tides") represents the effect of the non-conservative force. After some algebra, the energy evolution equation yields

$$m_p \frac{d\mathcal{L}_p}{dt} + \frac{j_p}{j_1} m_1 \frac{d\mathcal{L}_1}{dt} = m_p \left(\frac{d\mathcal{L}_p}{dt}\right)_T + \frac{j_p}{j_1} m_1 \left(\frac{d\mathcal{L}_1}{dt}\right)_T. \tag{A4}$$

Combining this with equation (A2) we find

$$\frac{d\ln\mathcal{L}_1}{dt} = \frac{d\ln\mathcal{L}_p}{dt} = \frac{1}{1+(m_1/m_p)\,(j_p/j_1)^{2/3}} \left[\left(\frac{d\ln\mathcal{L}_p}{dt}\right)_T + \frac{m_1}{m_p}\left(\frac{j_p}{j_1}\right)^{2/3}\left(\frac{d\ln\mathcal{L}_1}{dt}\right)_T\right] \tag{A5}$$

Next we examine the angular momentum

$$L = m_1\sqrt{GM_* a_1(1-e_1)^2} + m_p\sqrt{GM_* a_p(1-e_p)^2}, \tag{A6}$$

which evolves according to

$$\frac{dL}{dt} = \left(\frac{dL_1}{dt}\right)_T + \left(\frac{dLp}{dt}\right)_T. \tag{A7}$$

Introducing the auxillary variables $Y_i = \sqrt{1-e_i^2}$ we find

$$\begin{aligned} &\left[\frac{dY_1}{dt} - \left(\frac{dY_1}{dt}\right)_T\right] + \frac{m_p}{m_1}\left(\frac{j_p}{j_1}\right)^{1/3}\left[\frac{dY_p}{dt} - \left(\frac{dY_p}{dt}\right)_T\right] \\ &\quad = Y_1\left[\left(\frac{d\ln\mathcal{L}_1}{dt}\right)_T - \frac{d\ln\mathcal{L}_1}{dt}\right] + \frac{m_p}{m_1}\left(\frac{j_p}{j_1}\right)^{1/3} Y_p\left[\left(\frac{d\ln\mathcal{L}_p}{dt}\right)_T - \frac{d\ln\mathcal{L}_p}{dt}\right]. \end{aligned} \tag{A8}$$

Combining equations (A5) and (A8) gives (Gomes 1998)

$$\begin{aligned} &\left[\frac{dY_1}{dt} - \left(\frac{dY_1}{dt}\right)_T\right] + \frac{m_p}{m_1}\left(\frac{j_p}{j_1}\right)^{1/3}\left[\frac{dY_p}{dt} - \left(\frac{dY_p}{dt}\right)_T\right] \\ &\quad = \left\{\left(\frac{d\ln\mathcal{L}_1}{dt}\right)_T - \left(\frac{d\ln\mathcal{L}_p}{dt}\right)_T\right\}\left[Y_1 - \frac{j_p}{j_1}Y_p\right] \Big/ \left[1 + \frac{m_1}{m_p}\left(\frac{j_p}{j_1}\right)^{2/3}\right]. \end{aligned} \tag{A9}$$

Writing this in terms of e_i and a_i, we find

$$\frac{e_1}{\sqrt{1-e_1^2}}\left[\frac{de_1}{dt}-\left(\frac{de_1}{dt}\right)_T\right]+\frac{m_p}{m_1}\left(\frac{j_p}{j_1}\right)^{1/3}\frac{e_p}{\sqrt{1-{e_p}^2}}\left[\frac{de_p}{dt}-\left(\frac{de_p}{dt}\right)_T\right]$$
$$=\left\{\left(\frac{d\ln a_1}{dt}\right)_T-\left(\frac{d\ln a_p}{dt}\right)_T\right\}\left[\frac{j_p}{j_1}\sqrt{1-{e_p}^2}-\sqrt{1-e_1^2}\right]\Bigg/2\left[1+\frac{m_1}{m_p}\left(\frac{j_p}{j_1}\right)^{2/3}\right] \quad \text{(A10)}$$

Support for this work was provided by NSERC of Canada.

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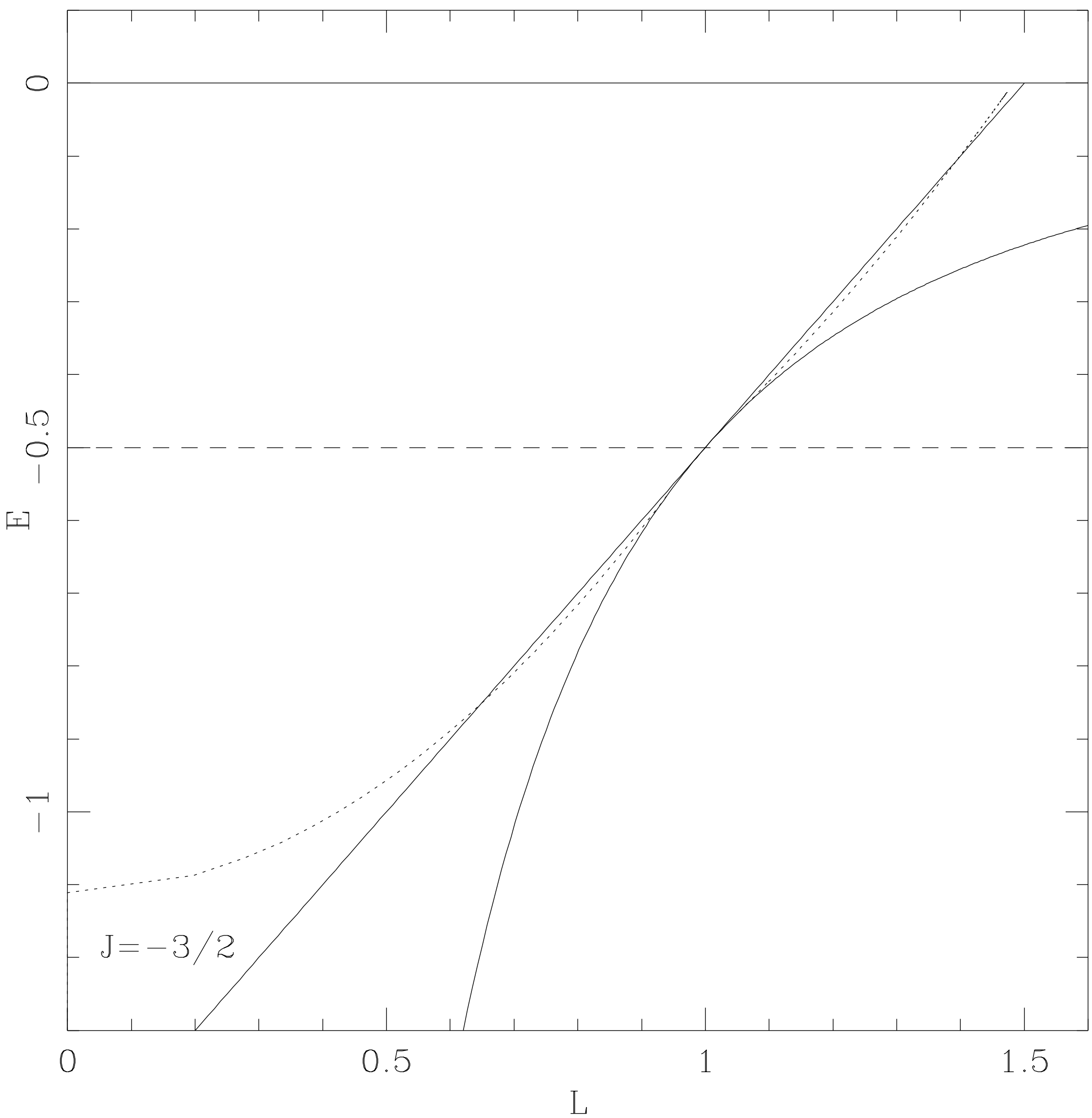

Fig. 1.— The $E - L$ plane for a planet with $e_p = 0.1$. The curved solid line is the maximum angular momentum possible for a body of the given energy; it corresponds to a circular orbit ($e = 0$). The dotted line corresponds to a planetesimal orbit that just grazes the orbit of the Jupiter mass body. The solid line labeled $J = -3/2$ illustrates the minimum value of the Jacoby parameter that must be reached by the asteroid in order to be lifted from an orbit with $a < a_p$ to an orbit with $a > a_p$. In this figure we employ units in which $GM_* = a_p = 1$.

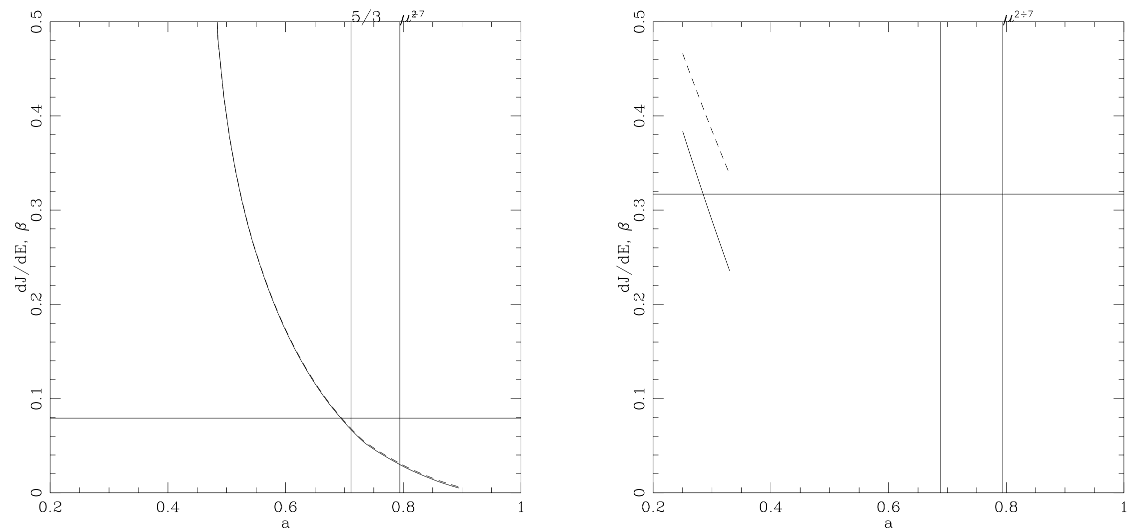

Fig. 2.— a) The time averaged change dJ/dE for a planetesimal with initial eccentricity $e = 0.05$, as a function of its initial a (dashed curve), and the corresponding β (solid curve). The planet is assumed to have a mass equal to that of Jupiter ($318 M_\oplus$), and $e_p = 0.05$. The solid vertical line marked $\mu^{2/7}$ marks the region where first order mean motion resonances overlap, producing large scale chaos. The solid vertical line near $a = 0.71$ marks the location of the 5/3 mean motion resonance. Note that $dJ/dE \approx 0.03$ near the $\mu^{2/7}$ region. The horizontal line corresponds to γ for an inner planet of mass $10 M_\oplus$ trapped in the 4/1 mean motion resonance. b) Same as in (a), except that $e_p = e = 0.5$. The horizontal line corresponds to an inner planet with a mass of $40 M_\oplus$.

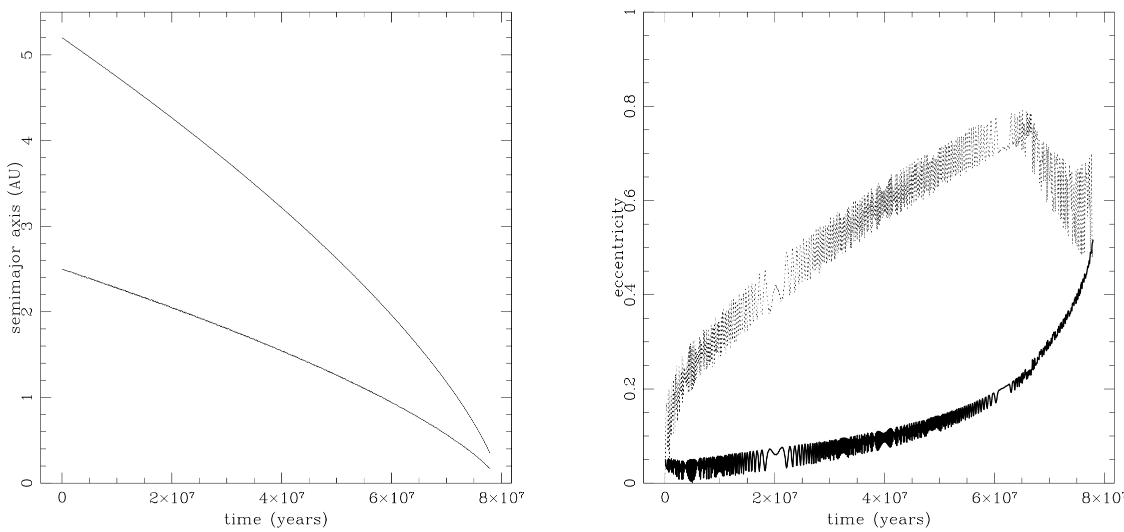

Fig. 3.— The evolution of a system of 2 bodies, one having a mass of $40M_{\oplus}$, placed inside the orbit of the second, a Jupiter mass object that is forced to migrate inward. The inner body is placed just inside the 3/1 mean motion resonance. Panel (a) shows the semimajor axis of both bodies as a function of time, while (b) shows the eccentricity; the light line (higher e) corresponds to the inner, lighter planet.

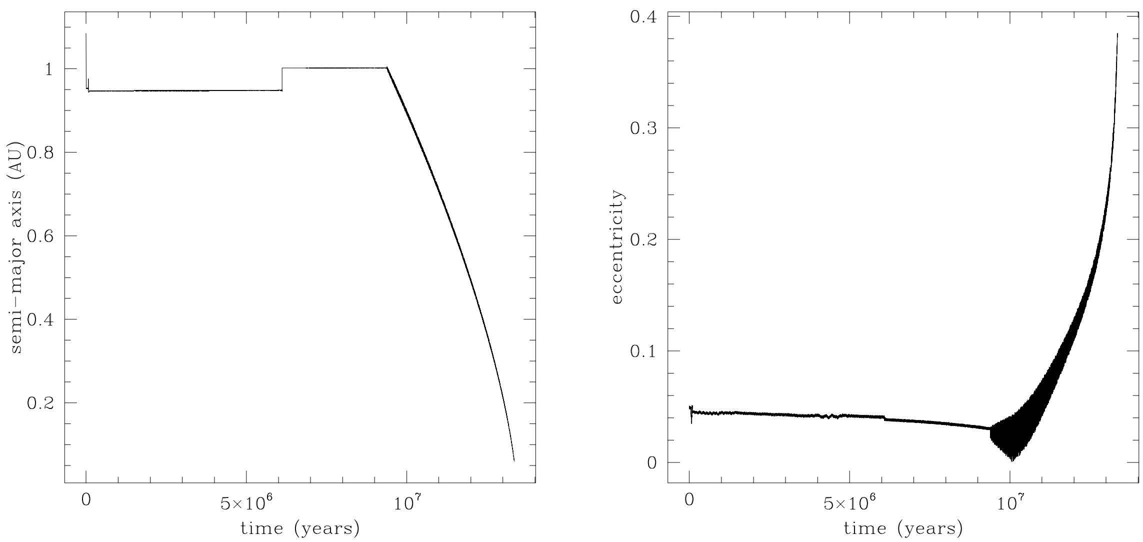

Fig. 4.— The evolution of a system of 5 bodies with masses randomly distributed between 0.3 and $10M_{\oplus}$, placed inside the orbit of a Jupiter mass object that is forced to migrate inward. Three of the small planets merged with each other to create a $6.3M_{\oplus}$ mass planet. One of the small planets merged with the Jupiter mass planet, while the fifth small planet was ejected. Panel (a) shows the semimajor axis of one of the low mass planets that merged to form the $6.3M_{\oplus}$ mass body; the mass increases with time due to collisions with other bodies, seen as jumps in a. This body was caught into resonance with the Jupiter mass planet at $t = 9 \times 10^6$ years. (b) The eccentricity of the Jupiter mass body as a function of time. It damps slowly up until the time it captures the smaller mass body in (a), then rises rapidly. The value of β used in this run assumed that $e = 0.05$, which is not appropriate for the final $e_p \approx 0.4$.